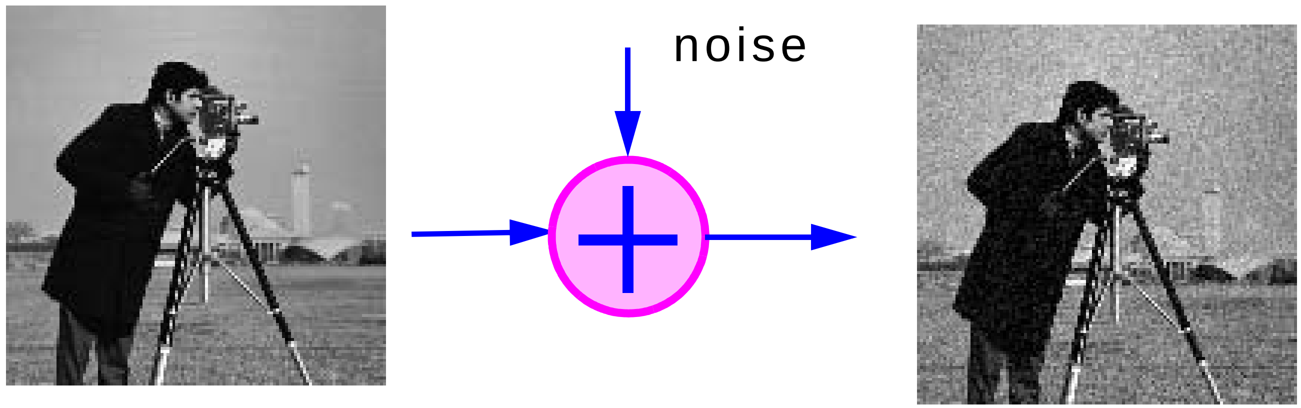
Applications in Denoising

Wavelet based denoising

Problem description



original image — observed image

Standard approach: Wiener filter has limitations

- blur, edge artifacts
- does not denoise well
- need choice of window

Wavelet approach

- works for images that are nongaussian (like they are!)
- simple nonlinear technique

Wavelet denoising...
... basic idea

Introduction

- simple technique [Donoho et al]
- nonlinear thresholding algorithm
- soft thresholding: shrinkage
- hard-thresholding: keep or kill

Wavelet transform

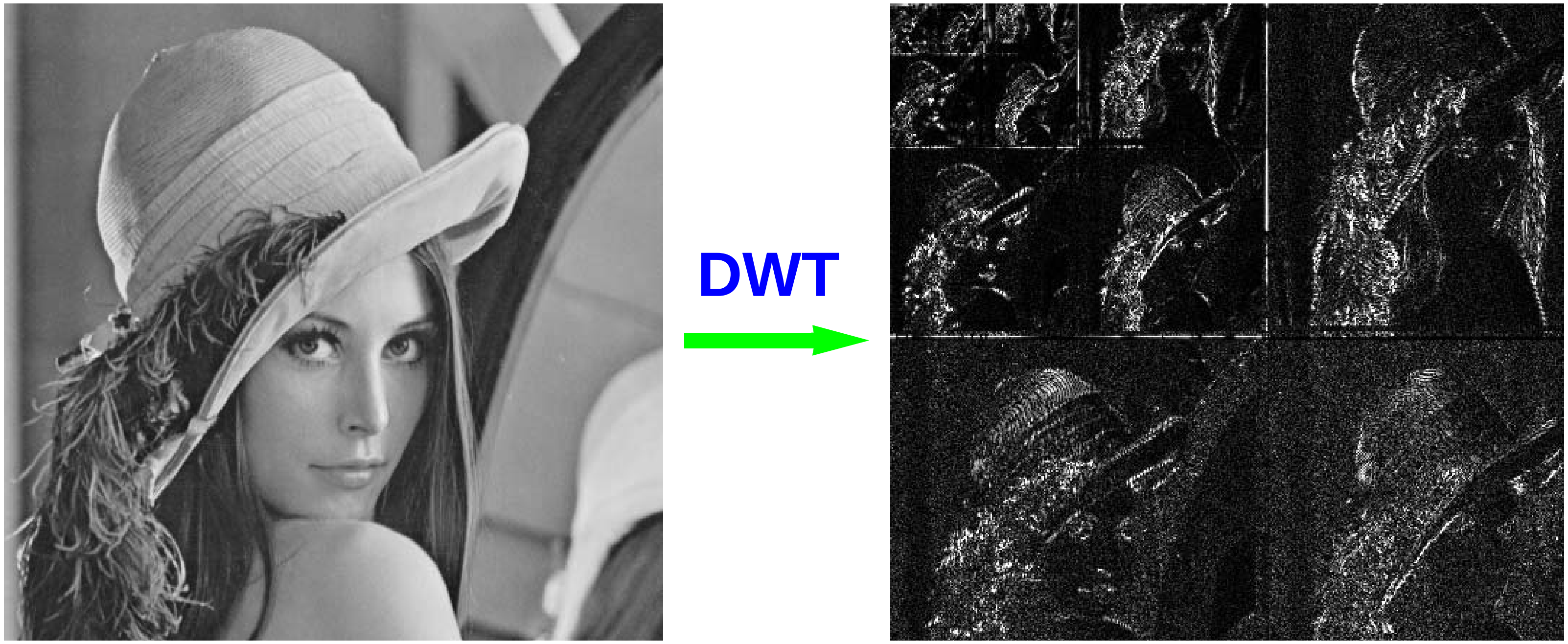
- natural decomposition
- other possible expansions

Threshold choice

- difficult
- minimax results, but T depends of sample size
- in practice, heuristic, independent of sample size

Wavelet denoising...
... basic idea

Typical images have varying local energy levels



- large threshold removes noise,
  small threshold retains details
- edges and textures tolerate noise, not blur;
  opposite for smooth areas.

Spatial and scale adaptive thresholds based on signal energy

Wavelet denoising algorithm

A. Take a wavelet transform

- standard orthonormal expansion
- possibly overcomplete
- possibly repeat algorithm for many shifts

B. Apply thresholding to wavelet coefficients

- in each band,
  choose τ of the order of the noise standard deviation
- decide if hard or soft thresholding

C. Reconstruct signal with inverse wavelet transform

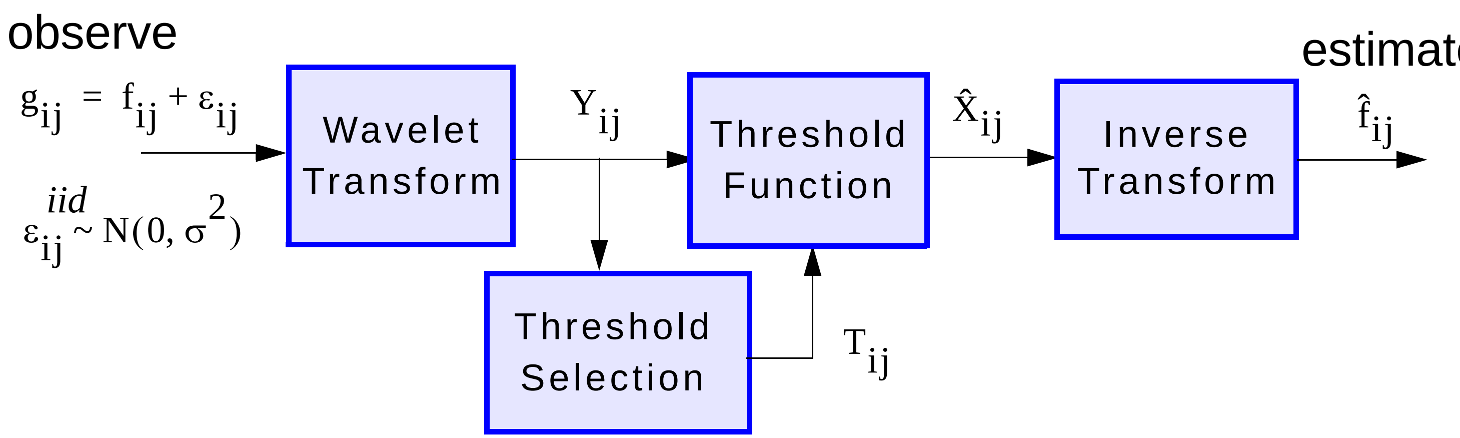
- orthonormal case: immediate
- averaging if multiple denoising have been made

Result:

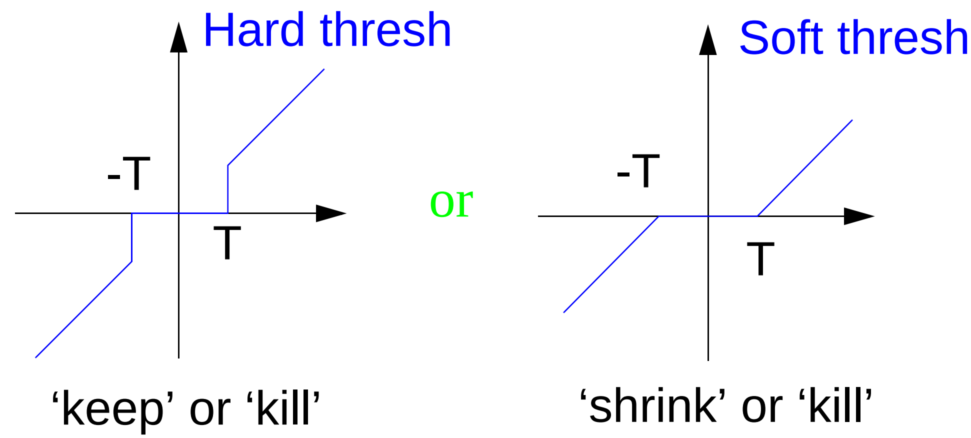
- low complexity, $O(N)$ algorithm
- can outperform standard methods (such as Wiener)
- nonlinear but simple
- possible to prove hard results (such as minimax, Donoho et al)

Wavelet thresholding

[Donoho,Johnstone94]



Threshold functions

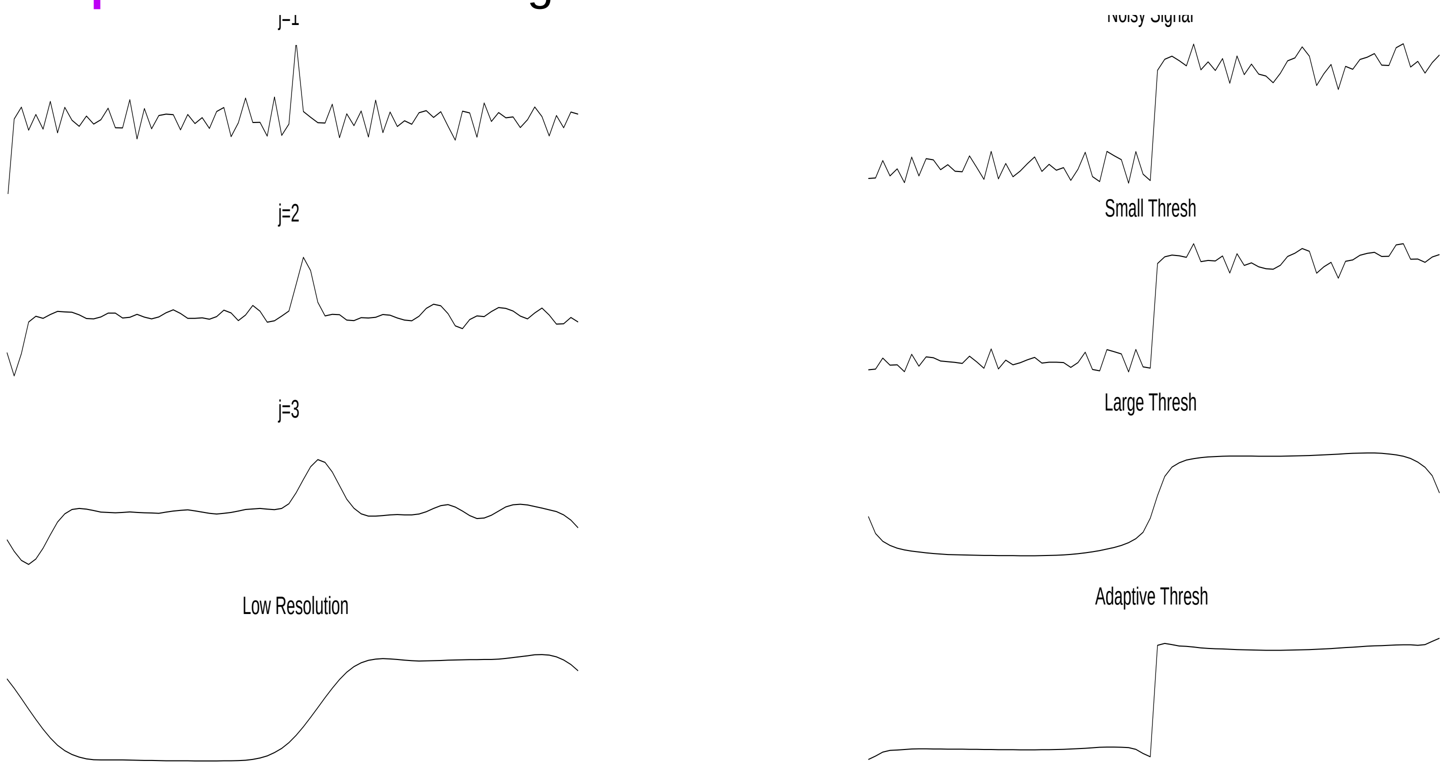


Adaptive wavelet thresholding

Motivation for adaptive thresholding

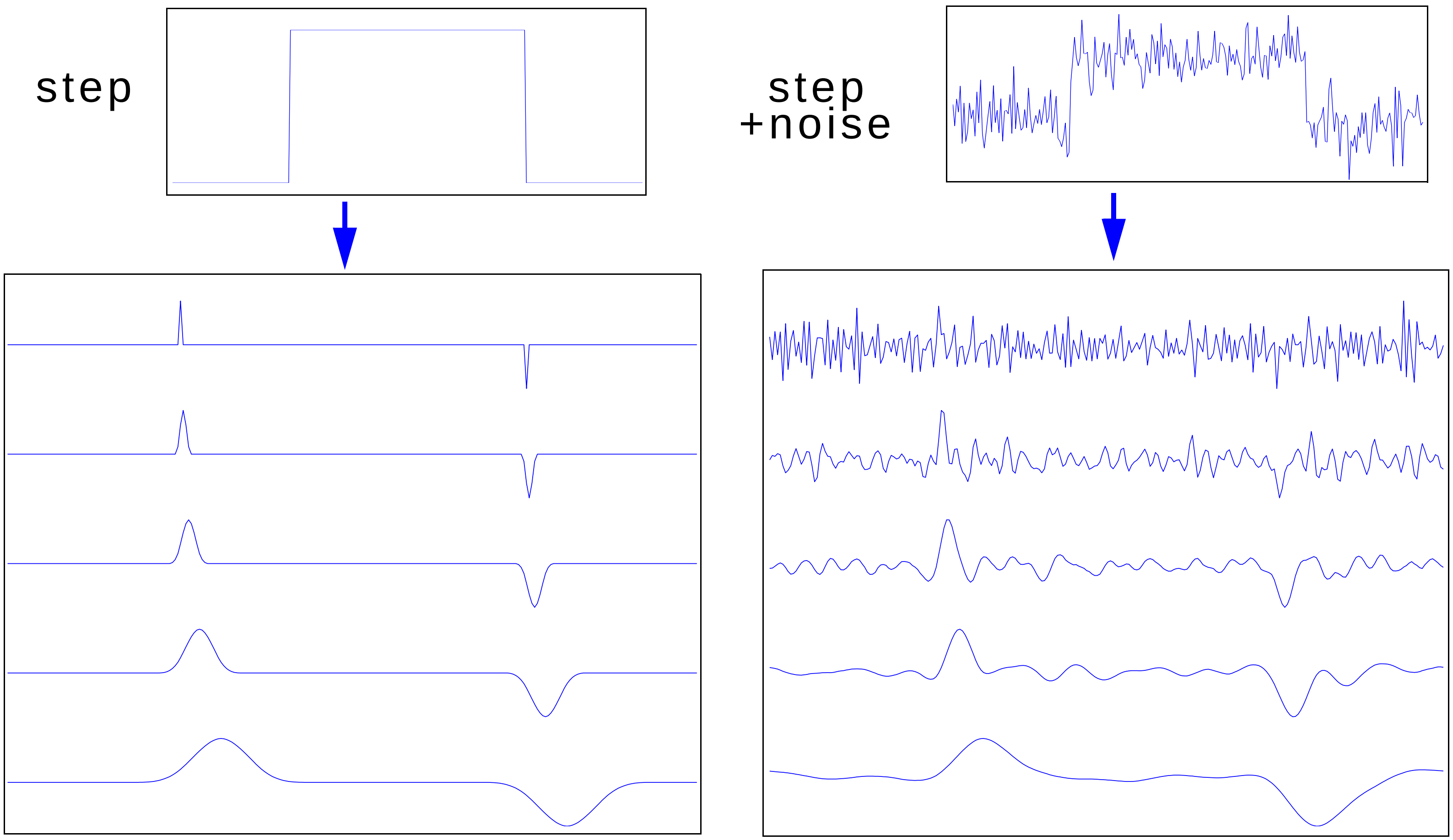
- large threshold removes noise,
  small threshold retains sharpness
  $\Rightarrow$ best is to adapt based on singular or smooth regions
- spatial and scale-adaptive thresholding is easy

Example: WT of an edge



Spatial-adaptive thresholding

Spatial & scale adaptivity easy to achieve (change thresholds)



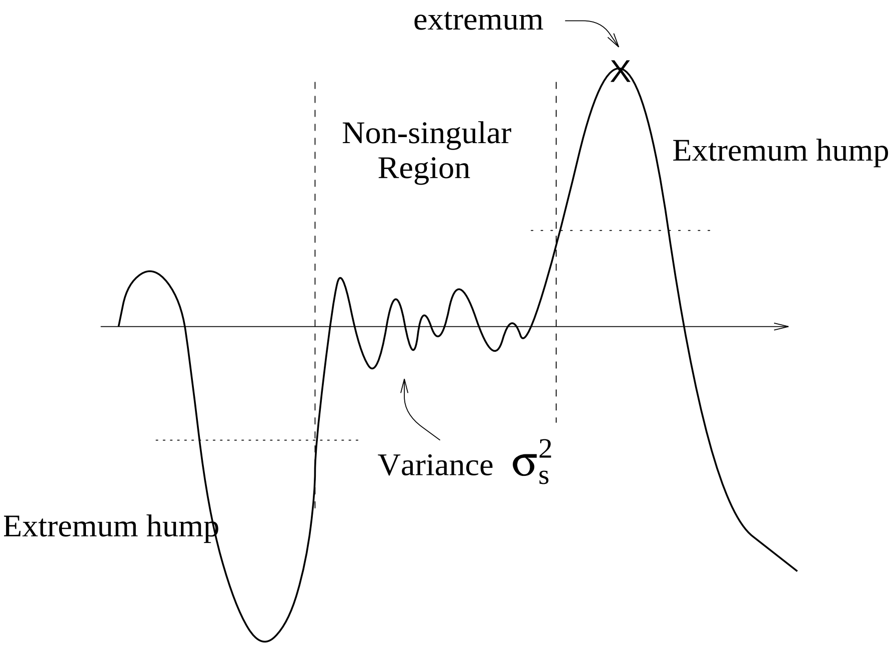
wavelet coefficients

Adaptive wavelet thresholding

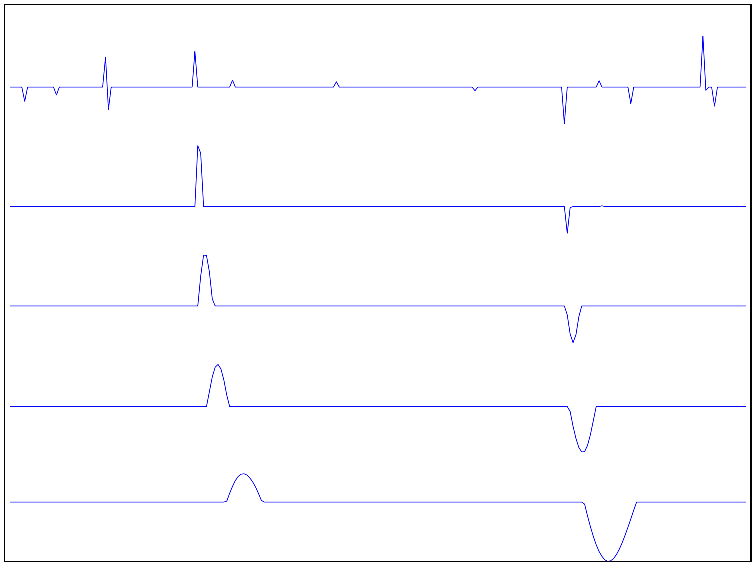
A. **Detect important singularities by associating extrema across the scales**

B. **At each scale, threshold between consecutive extrema humps to achieve denoising without blurring the singularities**

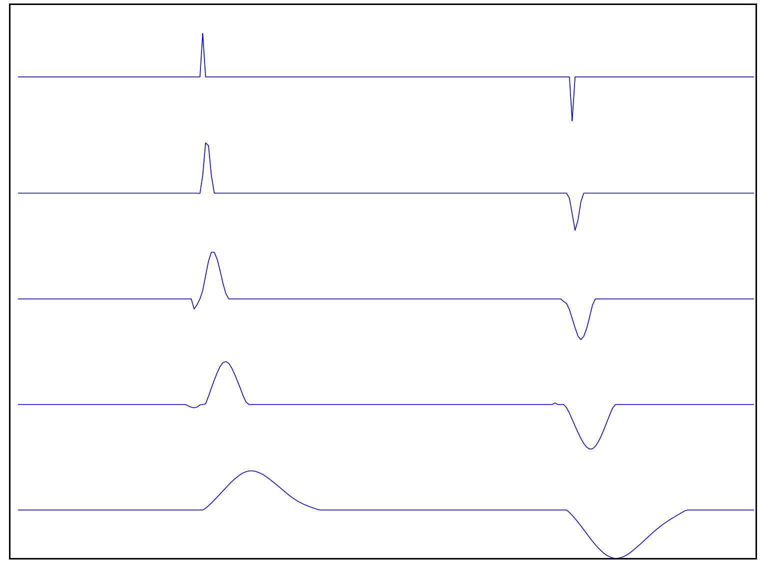
C. **When noise power becomes too large, it is necessary to threshold extrema humps at least in the finest scale**



uniform threshold

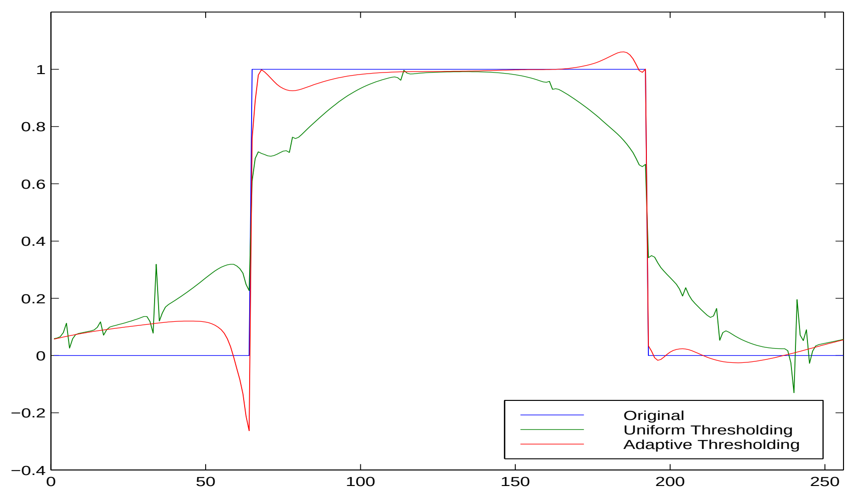


spatial adaptive thresholds



compare estimates:

mse(noisy)=0.073
mse(unif)=0.027
mse(adapt)=0.004



original

noisy, σ=22.5
mse=506

DWT adapt.
mse=66.4

nonsub. Oracle unif.
mse=55.2

nonsub. adapt
mse=51.9

DWT Oracle unif.
mse=66.6

original

noisy, $\sigma=25$
mse=625

DWT adapt.
mse=141

nonsub. Oracle unif.
mse=127

nonsub. adapt
mse=109

DWT Oracle unif.
mse=153